Mathematics, Innate Knowledge and Neuroscience

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Abstract:
After reviewing the position of Plato for the existence of innate knowledge, tabula inscripta, and the opposing position of Aristotle and Locke that the human mind is a tabula rasa, certain philosophical positions of the Ancient Greeks about Mathematics, will be recalled. The above positions will then be discussed within the context of the astounding modern achievements of Neuroscience. In particular, 'bottom up' processes which are basically innate, and 'top down' processes which rely on experience stored in memory, will be discussed. In addition, the importance of different time scales in conscious and unconscious processes will be elucidated. Finally, it will be shown that modern brain imaging techniques provide an important example of the crucial role played by Mathematics in understanding the essence of various phenomena, a fact which was philosophically appreciated first by Pythagoras.

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1. Introduction

It is difficult to find a scientific field in which Hellenic thought did not have a decisive impact. For example, the work of Thucydides provides a monumental reference to political science that remains a necessary tool for understanding international relations to this day. Democritus, the Epicureans and Heraclitus ("everything is in a state of flux", τα πάντα ρεί) introduced the basic elements of dialectic materialism. The assertion of Epictetus that "sadness is caused not by facts themselves, but by our opinion about them" foreshadows cognitive psychology. Is there a more important text on Democracy than the *Funeral Oration*, attributed by Thucydides to Pericles?

Ancient Greek thought was not created in vacuum. The Neoplatonic philosophers Porphyry, Iamblichus, Hierocles and Proclus (third century A.D.), have presented a detailed analysis of how the 'ancients', namely the Egyptians, the Chaldeans and the Phoenicians, influenced Pythagoras and Plato. However, Plato himself states that whatever the Greeks learned from the ancients they were able to make it complete (ότι περ αν Έλληνες βαρβάρων παραλάβωσι κάλλιον τούτον εις τέλος απεργάζονται).

Perhaps the main difference of the Greeks with their prestigious creditors is that the Greeks begun the fundamental process of 'thinking about thinking'. They realised that the search for understanding requires appropriate methodologies so that one can reach the hidden, 'το *ἄδηλον'*. The necessity of such methodologies was summed up in the dictum proffered by Anaxogoras and strongly endorsed by Democritus: "phenomena are the sight of the hidden" (όψις γάρ τών αδήλων τά φαινόμενα). Already in *Theaetetus* (Θεαίτητος) Plato attempts to answer the question "what is knowledge?" He suggests identifying knowledge with "true belief accompanied by logos", but with remarkable honesty he has the dialogue end in failure because, despite several attempts, the interlocutors do not succeed in defining *logos*'. Aristotle, in his *Posterior Analytics* states: "we have knowledge of a fact when we know (a) the cause from which this fact results and (b) that this fact cannot be otherwise than it is". However, both Plato and Aristotle understood that if there were no knowledge other than demonstrative knowledge, then we would end up in an infinite regression. Indeed, to know something, we have to be able to prove it on the basis of something else which we have to know, which in turn we must be able to prove in terms of something else, etc. In a sense, both Plato and Aristotle overcome this problem by postulating 'a beginning'. In *Parmenides*, Plato states "there is only one beginning, there cannot be many". On the other hand, Aristotle introduced the idea that there exist certain principles which are known to humans

intuitively. The above positions of Plato and Aristotle are closely related with their relevant positions on innate knowledge to which we will return later. We conclude this introduction by emphasizing that the Greeks pursued the search for knowledge with such a passion that justifies Aristotle's claim that humans by nature "desire knowledge for itself, seeking no benefit beyond the pleasure that this search brings". This intellectual and moral framework of the search for 'truth' opened up what Immanuel Kant called "the sure path of science", first demonstrated clearly in Mathematics.

2. Mathematics

For the Greeks, Geometry is not the manipulation of figures in physical constructions, but the understanding of their properties in pure thought. The transition from praxis (techniques for dealing with practical activities) to theoria (form of pure knowledge) occurred for the first time in human history in the treatment of Mathematics by the Greeks.

It is well known that the enormous contribution of Mathematics to our culture stems from two basic facts: First, Mathematics facilitates the emergence of a logical way of thinking. It is interesting that Darwin expressed his regret for not having studied Mathematics, as according to him, this would have given him a sixth sense. Second, the fundamental laws of nature can be expressed in a mathematical language; hence, the mathematical analysis of these laws leads to the deepest possible understanding of the essence of physical phenomena. Both of these facts were already appreciated by the Greeks.

Regarding the first fact, namely, the importance of mathematics in the development of a logical way of thinking, the crucial role assigned to Mathematics by Plato becomes clear by recalling the main sign on the entrance of the Academy of Athens: "No one can enter who does not know geometry" (Μηδείς αγεωμέτρητως εισήτω). According to Plato in the *Republic* Mathematics is "conducive to the awakening of thought", and according to the Neoplatonic philosophers, Mathematics helps us understand the realm of ideals by analogy. Proclus writes: "Plato explained to us many wonderful doctrines about the gods by means of mathematical forms". Iamblichus states: "if one wonders how the many could be in the One, let him think of the monad". Furthermore, Mathematics was the basic paradigm used by Aristotle for the development of his logic. Indeed, the essence of Aristotle's logic is the doctrine of

'inference' (συλλογισμός), via which it is possible to distinguish between correct and incorrect types of argumentation. However, inference had already played a decisive role in producing some startling results in pre-Euclidean geometry.

Regarding the second fact, namely, the role of Mathematics in understanding nature, Bernard Russell considered Pythagoras one of the greatest intellectuals of all times, precisely because Pythogoras was the first to appreciate that nature can be understood via pure thought in general and via Mathematics in particular. According to him "everything is made of numbers". Galileo is usually credited for understanding that the physical laws are written in a mathematical knowledge. However, Iamblichus has already stated: "The shapes created and the forces which exist between them, as well as the illuminations of the moon and the order of the spheres and the distances between them and the centres of the circles on which they move, everything is expressed in numbers". In addition, Proclus wonders: "How is the sensible world organized? According to what principles? What principles was it born from, if not from mathematical ones"? According to Pythagoras, not only physics, but also Ethics borrows notions from Mathematics. For example, the mathematical notion of mean leads to the notion of moral virtue as the mean of two extremes, namely excess and deficiency (justice is symbolized by the number 5, which is the mean of 1 and 9).

It is interesting that Plato placed Mathematics in an intermediate reality (διάνοια) between his two basic realities, (a) the intelligible (νοητός κόσμος) and (b) the sensible (αισθητός κόσμος). Recall that in each of these realities there exist corresponding forms: the intelligible or ideal forms are created by Gods (παρά Θεοίς). In the intermediate reality, where Mathematics lives, there exist the discursive forms which are projections of the ideal forms. In this sense, Mathematics functions as a bridge: On the one hand, it tends towards perfection since it illustrates in a paradigmatic way characteristics of perfect forms, but on the other hand it gets 'contaminated' by dealing with reality, as for example with the calculation of the orbits of the stars (it should be recalled that according to Platonism the more sensible something is, the less significant it is). For Plato, Mathematics is incomplete because it does not examine the principles (axioms) from which it derives; these axioms can be examined only by Dialectis, the only perfect science. However, Proclus believed that the reliance of mathematics on axioms should not be considered as a weakness, since axioms are innate in souls. According to him, the existence of axioms is consistent with the position of Stoic philosophers for the existence of universal truths which are evident to everyone, as well as with Aristotle's

position (mentioned earlier) about the existence of certain truths which cannot be proven, but which are intuitively evident. This qualification of axioms provides a firm scientific basis to mathematics: starting with innate truths and using Aristotelian logic, mathematics derives rigorous results.

Certain philosophers tried to elevate mathematics to an even higher level than that of Plato. Especially Nicomachus, 2nd century B.C., the author of Introduction to *Arithmetics* as well as of *Theologoumena*, after examining every number from a physical, ethical and theological point of view, reaches the conclusion that numbers are not merely projections of perfect forms, but that their characteristics are precisely those of gods. Also, Iamblichus, in his ten famous books on *Pythagorism*, tried to mathematize the Platonic theory; according to him, since the sensible world is organized on the basis of numbers, mathematics not only subsumes physics and ethics, but also foreshadows Dialectics. Finally, Proclus tried to elevate geometry to a level similar to the one that Nicomachus had elevated arithmetic; according to him, geometry, due to its images and syllogisms is more suitable than arithmetic to teach eternal truths to the fallen souls.

Concluding the section on Mathematics it is worth noting that Euclid's Elements written in 300 B.C. is perhaps the greatest scientific achievement of antiquity. It is difficult to find another work that has had a stronger impact on the development of modern physics and mathematics than this monumental work. Several modern intellectual giants, including Kepler, Descartes and Newton, studied Euclid's geometry in detail. Newton's differential calculus was formulated in a geometrical language precisely because he believed that a work is scientific only if it is written in the rigorous geometrical language of Euclid's Elements.

3. Innate Knowledge

The speed and accuracy of learning a vocabulary, as well as the existence of universal grammatical structures, led Chomsky to the conclusion that there exists an innate universal grammar. Recent studies involving children of the Munduruku indigenous group have shown that basic geometrical concepts as well as the ability of approximate (but not exact) arithmetic are also innate. The Munduruku, who live in an isolated area of the Amazon, do not have words for basic geometrical concepts such as parallel or symmetric, and also do not possess maps or other relevant tools that facilitate the development of geometrical intuition. Also, they have words only for numbers from 1 to 5. In one of the relevant experiments, children were presented with 1 to 25 dots and

were asked to find their number. From 1 to 3 dots the answer was precise. But from 3 to 25 dots the answer was approximate. For example, despite the fact that there exists a word for 5, for 5 dots among the answers were: 3, 4, 5, and about as many as the fingers of one hand. For 13 dots, among the answers was two hands and something more. Other similar experiments demonstrated the ability of the children for addition and subtraction, but, again, approximately. It seems that for precise arithmetic for numbers higher than 3, one requires the existence of a specific algorithm, as well as the existence of a specific word or at least of an abstract symbol for each number.

Does there exist innate knowledge and if yes, what is its origin? The debate of this fundamental question began with the Greeks. Plato and later Leibnitz and Kant emphasized the existence of a-priori knowledge. On the basis of the platonic dialogue *Phaedon* and of the explanation of this dialogue by the Neoplatonic philosopher Syrianos, there exist several types of souls. But even the corrupted souls were earlier pure (άχραντες), thus they contain in themselves a priori knowledge. For this reason, the process of learning and discovering is merely a process of recollection. By stimulating these souls with appropriate questions, the teacher stimulates this process of recollection. Within this framework, the maieutic method of Socrates acquires an absolutely necessary character. After having led with his questions a slave to discover certain geometrical truths, Socrates exclaims: "The slave has always had this knowledge within his soul".

In a sense, the science of Dialectics deals precisely with the ideal knowledge of the pure souls. To this idealistic position, Aristotle presented an undeniably, empiricist reply. In *Posterior Analytics*, he grants full scientific status to several disciplines, including astronomy, acoustics, harmonics and optics. Each of these sciences has it's own unprovable principles. These principles become known to humans at the end of an inductive process which involves perception, memory, experience and the mind, νους. For Aristotle, νους is by definition the cognitive state that comprehends principles. Hence, the combination of νους and of demonstrative knowledge leads to the truth. In his analysis, Aristotle also emphasizes that learning is impossible without associating ideas. Following Aristotle, the British empiricist John Locke proposed that the human mind is a *tabula rasa*, i.e. a blank slate, which is inscribed as a result of experience.

It appears that for the first time in the history of mankind, there now exists a scientific framework for the deep study of such questions: It is well known that Ivan Pavlov and Edward Thorndike provided experimental proof that there exists a particular

type of learning which is based on associating different stimuli. Their classical experiments led to the emergence of the rigorous empirical school of behaviourism. Indeed, it was clearly argued by J.B. Watson and B.F. Skinner that behaviour could be studied experimentally as rigorously as phenomena in the physical sciences. This led to deeper understanding of the process of acquiring knowledge. For example, Leon Kamin in 1969 demonstrated that animals do not simply learn that a neutral stimulus (such as the bell ring) precedes a reward (such as food), but rather that the stimulus predicts the reward. This suggests that associative learning is based on the ability of the brain to couple events that occur together regularly, so that it can predict the occurrence of an outcome. The behaviourists, following Sigmund Freud, deliberately avoided any speculation about the relation of behaviour with brain activity and focused exclusively on observable behaviour.

The above arguments are consistent with Aristotle's ideas. However, when the understanding of how the brain works reached the level that a process begun of correlating observed behaviour with neuronal activity, it became clear that the Platonic position of *tabula inscripta* is also correct. Indeed, the complicated neuronal circuits of the 100 billion neurons of the brain, together with the astounding dynamic behaviour of their synapses, provide the material basic for the existence of both elemental innate knowledge, as well as a predetermined predisposition for learning.

One of the great challenges of the modern neuroscience is precisely to delineate the neuronal mechanisms involved in the interplay between innate and acquired knowledge. In the remaining of this lecture, some of these mechanisms will be reviewed and also the relevant role of Mathematics in deciphering these mechanisms will be mentioned. However, please allow me to first open a parenthesis: The explosion of the cultural and scientific achievements that occurred at Fin de Siecle in Vienna, provide perhaps the closest analogue in modern times to the monumental achievements of ancient Athens. The existence of Kurt Godel and Ludwig Wittgenstein, Otto Wagner and Walter Gropius, Arthur Schnitzler and Hugo von Hofmansthal, Carl von Rokitansky and Josef Skoda, Sigmund Freud, Gustav Klimt and Oscar Kokoschka, Gustav Mahler and Arnold Schoenberg, remind us that it is possible for a single city, in a short period of time, to witness unprecedented achievements in such diverse areas as philosophy, architecture, literature, medicine, psychology, painting and music. Perhaps it is worth searching for similarities between these two great epochs. Here, it is only noted that Athens and Vienna were two relatively small cities with a social structure that allowed

intellectuals of different disciplines to mix and to exchange ideas. Also, the gymnasiums of both cities did <u>not</u> distinguish between what we would call today sciences, humanities and arts. This suggests that our current view of the advantage of interdisciplinary research, should perhaps become even broader, so that not only boundaries between different sciences are eliminated, but also a unification is achieved between sciences, humanities and arts.

4. The Process of Learning at the Molecular Level

Let us first review some basic elements of neuroscience. The fundamental functional unit of the nervous system is the neuron. A typical neuron consists of (a) the cell body which contains the nucleus, (b) about 1000 dendrites which receive information from other cells, and (c) the axon, which transmits information to other cells. This information is coded in the form of electrical signals. These signals propagate along the axon with the slow speed of 30 meters per second (recall that the speed of propagation along metals is close to the speed of light), however, they have the important advantage that they propagate unchanged, conserving their height and width. Such a signal, called an <u>action potential</u>, is a nonlinear phenomenon, a kind of a soliton. This nonlinearity is due to the existence of tiny ionic gates across the neuronal membrane which allow the flow of ionic current, giving rise to a nonlinear conductivity. The non-linear nature of the action potential implies that this phenomenon is of the type of all or nothing, i.e., a weak stimulus does not generate an action potential, but if the stimulus is above a certain threshold, then an action potential is generated near the cell body and propagates to the presynaptic end, see diagram 1. There, it causes the release into the synaptic cleft of a chemical substance called <u>neurotransmitter</u>. Neurotransmitters stimulate the receptors of the postsynaptic neuron and in this way the chemical message is transformed back into an electrical one.

Neurons are highly specialized. For example, sensory visual neurons respond only to light, whereas sensory hearing neurons respond only to acoustic waves. In the cerebral cortex, there exists an even higher level of specialization. For example, some cells respond only to vertical bars of light, whereas others respond only to horizontal bars; these orientation cells are topographically organized in certain cellular columns in the domain of the brain called V1. Similarly in the domain V4, there exist cells which recognize colour, and in the domain V5 there exist cells responsible for recognizing motion.
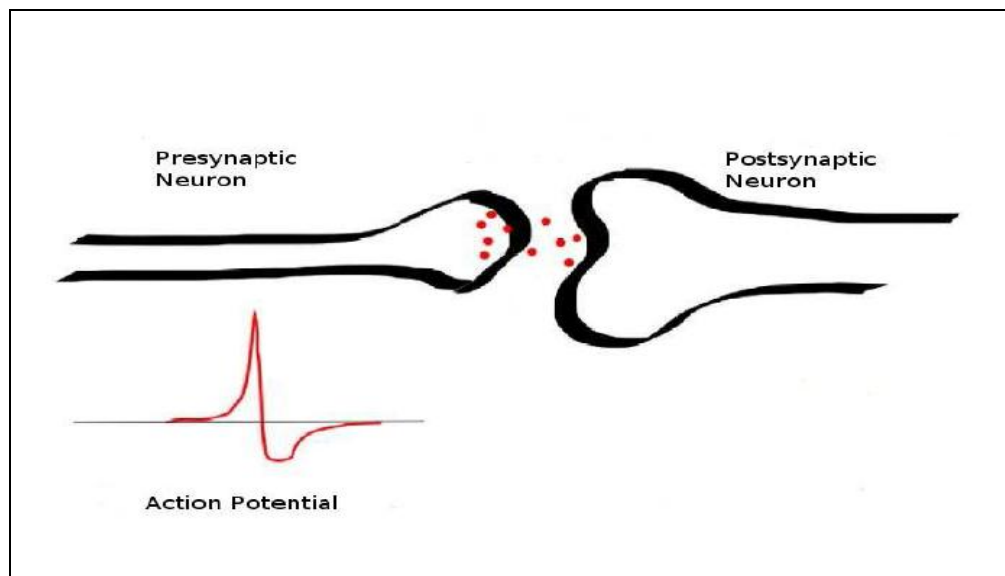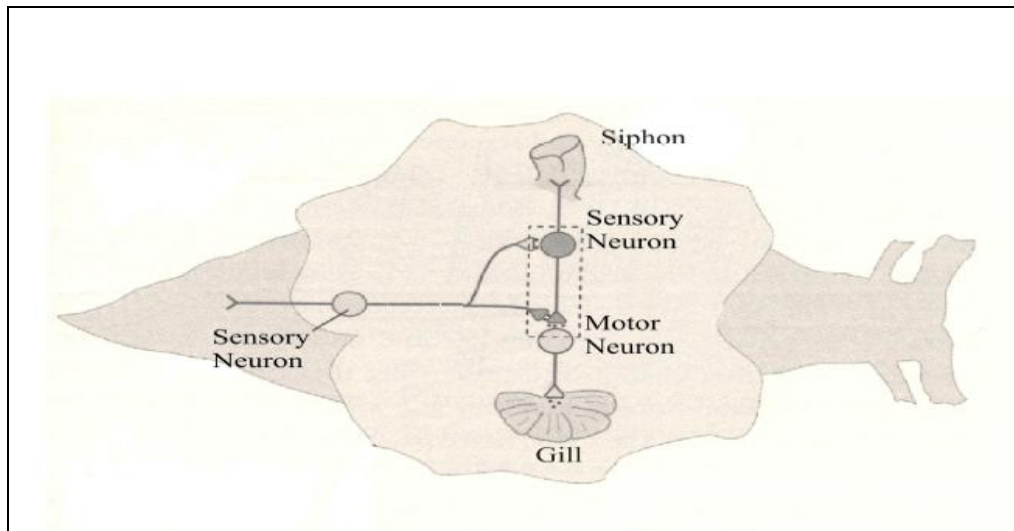
Diagram 1

The elucidation of the above mechanisms led to the awarding of several Nobel prizes, including: Cajal and Golgy, 1906, for the anatomy of neurons; Dale and Loewi, 1936 and Katz 1971 for the chemical theory of communication across synapses; Eccles, Hodgin and Huxley, 1963 for the existence of ionic current; Hubel and Wiesel 1981 for their studies in visual processing; Neher and Sackman 1991 for the measurement of the ionic current; Mac Kinnon 2003 for the structure of the ionic gates.

Let us now return to learning at the molecular level and summarize the remarkable studies of Eric Kandel, Nobel prize 2000, on the giant marine snail Aplysia. In order to breath, Aplysia uses an external organ called gill, which lies in a protected cavity. Weak tactile stimuli at the siphor make the gill to withdraw into the cavity. After a repetition of weak touches the snail becomes habituated and therefore its withdraw reflex diminishes. But, when the weak touch is paired with a shock to the tail, the snail is sensitized and then even a weak touch leads to a strong gill withdraw reflex. How does Aplysia learn to do this? The weak stimulus activates a sensory neuron and the action potential of this neuron causes the release of the excitatory neurotransmitter glutamate, which in turn stimulates the motor neuron responsible for the withdraw reflex, see Diagram 2.

Diagram 2

The shock at the tail stimulates another sensory neuron and the action potential of this neuron causes the release of another neurotransmitter called serotonin. This causes the release of a larger amount of glutamate and this in turn yields a stronger withdraw reflex. This behaviour lasts only for a few minutes. However, if instead of one shock there exist five shocks, then the changes at the neuronal level are truly dramatic. Namely, now the higher amount of serotonin, not only causes the release of more glutamate, but also it activates a particular gene which initiates the process of creating new synapses; hence now an anatomical change takes place and the behaviour of a stronger withdraw reflex lasts for weeks instead of minutes.

It is clear that the relevant neuronal architecture provides the basis for both the existence of the elemental innate knowledge possessed by Aplysia, namely that it knows to withdraw following a touch at the siphon, as well as its ability to learn from experience. The latter, more advanced form of learning, is achieved via the qualitative and quantitative changes at the level of synapses. If one could establish that similar mechanisms take place in the human brain, then one could argue that

Plato lives in the neuronal circuits, whereas Aristotle lives in the synapses.

The basic studies regarding the triptych neuron doctrine-ionic gates-chemical theory of synaptic transmission, were conducted in squids; the vision studies in monkeys and the sensitization studies in Aplysia. Is it possible to generalize the conclusions of these studies? Aplysia has only 20 thousand neurons, whereas the human brain has more than 100 billion. On the other hand, the theory of evolution suggests that nature uses

similar approaches for the solution of similar problems. Two examples of this principle are the following:

(a) The action potential provides the universal mechanism of propagating information in all neurons.

(b) The mouse uses mechanisms similar with those used by the Aplysia. In particular, during the process of memorizing a particular place, it is possible to observe in the hippocampus of the mouse quantitative changes of the relevant synapses. However, long term memory requires the creation of new synapses. Instead of serotonin the mouse uses dopamine, but it is interesting that the creation of new synapses involves the activation of the same gene as in the Aplysia, namely of a gene called CREB.

Basic neuronal mechanisms have indeed universal validity. However, the analysis of human learning also requires differentiating between conscious and subconscious processes.

5. Conscious versus Subconscious Learning

Let us first review (a) some historical facts, (b) elements of Freud's theory of mind and (c) elements of the functional compartmentalization of the brain.

(a) Historical remarks

The question of whether the control centre is the heart or the brain, was extensively debated by the Greeks. Alcmaeon (5th century B.C.E.), like Plato and Hippocrates after him (but not Aristotle) chose the brain. Alcmaeon may even have investigated with a probe the back of the eye in order to establish the connection of the eye with the brain. This is consistent with the fact that at the beginning, the Greeks identified knowledge with sense perception and especially with visual perception. This is also expressed in the Greek vocabulary where the most common verb for 'to know' is '*οιδα*', which comes from the same Indo-European root as the Latin '*videre'*, 'to see'. Herophilus and Erasistratus, who lived in Alexandria at the 3rd century B.C.E., carried out vivisections on humans (on criminals obtained out of prison by the kings according to Celsus); in this way they were able to distinguish sensory from motor nerves. Herophilus also made two more important contributions: First, by comparing the strength and the rate of different pulses and by making analogies with the harmonics of music, he was apparently the first to introduce the concept of *rhythm* into life sciences, and second, his extensive analysis

of dreams foreshadows the discovery of subconscious processes; this analysis won him praise from none other than Freud. However, in spite of these great advances, overall the situation with the nervous system was similar with other systems of Hellenistic medicine, namely there was substantial progress in anatomy, but function remained essentially unknown. Thus, the prevailing attitude in medicine remained the position of Galen (second century A.D.) who, like Hippocrates, argued that diseases are not caused by specific organ malfunctions, but by the imbalance of the four basic *humors*. The situation begun to change with the detailed anatomical dissections of Andreas Vesalius in the 1540s, with the discovery of the circulatory system by William Harvey in 1616, and decisively by the establishment of the discipline of Pathology by Giovanni Morgagni in 1750. In direct opposition to Galen, Morgagni argued that clinical symptoms arise from disorder of individual organs, and, echoing Anaxagoras, he suggested that "symptoms are the cries of suffering organs". His approach was fully integrated into clinical medicine through the extensive collaboration of the pathologist Rokitansky with the clinician Skoda.

(b) Elements of Freud's Theory

Freud was the first to attempt to develop a theory of mind with emphasis on the subconscious. His main claims were the following: First, most of our mental life, including most of our emotional life, is unconscious. Second, the life instinct (Ἔρως) and the death instinct (Θάνατος) dictate our innate instinctual behaviour. Third, mental illnesses represent exaggerated forms of normal mental processes. Furthermore, in collaboration with Josef Breuer, the leading Vienna internist in the treatment of hysteria, they established that unconscious mental conflicts can give rise to psychiatric symptoms, like hysteria, which actually can be alleviated by bringing the unconscious cause into patients conscious mind. Freud was influenced by the following:

1. The Darwinian revolution; in particular, Darwin in his last great book *The Expression of the Emotions in Man and Animals* in 1871, suggested that since humans evolved from simpler animals, humans must have similar instinctual drives with other animals.

2. The great nineteenth century physicist and physiologist Hermann von Helmholtz who had already established that unconscious processes are crucial for human visual perception.

3. The nineteenth century philosophers Friedrich Nietzsche and Arthur Schopenhauer who wrote explicitly about the importance of unconscious thinking, as well as of the power of subconscious drives.

4. The contemporary of Freud and Head of the Department of Psychiatry of the Vienna School of Medicine, Richard von Krafft-Ebing, considered the founder of the modern study of human sexual behaviour (he introduced the concepts of sadism, masochism and paedophilia).

5. His six-month fellowship with the great neurologist Jean-Martin Charcot, who in addition to his pioneering studies in amyotrophic lateral sclerosis and multiple sclerosis, was also an expert in hysteria and hypnosis.

6. His six years study at the laboratory of Ernst von Brücke, who was a close friend of Helmholtz and a strong opponent of vitalism.

It is interesting that although Freud was a student of the great psychiatrist Theodore Meynert, who tried to implement the Rokitansky-Skoda program to the brain, Freud decided not to pursue his original program of unifying the science of mind with the science of brain (a program pursued at about the same time by William James in USA), but instead decided to concentrate on a theory of mind.

(c) Functional Compartmentalization

The domains V1, V4, V5 of the cortex mentioned earlier, provide examples of functional compartmentalization, namely examples of the fact that cells performing a similar task are grouped together. The most dramatic confirmation of the fact that particular modules of the brain are involved in particular mental tasks, is due to studying the patient known as HM. In an attempt to treat the uncontrolled epileptic attacks of this patient, the inner surfaces of the temporal lobe and of the hippocampus were surgically removed. Following this surgery, this patient could still remember what occurred before the surgery and also could memorize for a few minutes. However, the patient lost completely the ability to convert short term memories into long term memories. For example, every time the patient met the treating psychiatrist, he greeted her like meeting her for the first time. Progress in delineating functional compartmentalization in humans was for many years relying on following patients with a particular neurological defect, and then localizing via brain autopsy the corresponding anatomical defect. The most dramatic discovery of this type is due to Pierre-Paul Broca, who first noted that damage in the left posterior frontal lobe yields the so-called Broca's aphasia, i.e., the inability of a patient to speak, although

the patient retains relatively intact the ability to comprehend; in 1864 Broca proclaimed: "we speak with the left hemisphere". Further progress was made by studying the effect of the activation of specific areas of the brain during neurosurgery.

Let us return to the basic question of human learning. We now know that complex mental behaviour is the result of the interaction of several specialized but also interconnected areas of the brain. How can we study in vivo such specialized areas? The new functional imaging techniques of positron emission tomography (PET), of single photon emission computed tomography (SPECT) and of function magnetic resonance imaging (MRI), offer us the fabulous opportunity to observe in vivo the activation of specific modules.

The recent amazing achievements in neuroscience imply that:

Every conscious experience is preceded by an unconscious phase;

hence, the process of awareness is a relatively slow process.

Let us be concrete: Benjamin Libet (see his book Mind Time, 2004), applied electrical stimulation to the surface of the primary somatosensory cortex, which is the module specializing in tactile sensation. This stimulation elicited a conscious sensation of localized tingling (the subject reported that these sensations were coming from the skin). But for awareness to occur it was necessary for the brief pulses of current to have duration for about 500 msecs (0.1 to 0.5 msecs in duration repeated at a frequency of 20-60 pulses per second). On the other hand, even a single weak electrical pulse to the skin produces a conscious sensation. Why this discrepancy? Actually, a single effective pulse to the skin induces a cortical activation called <u>evoked potential</u>. However, this does <u>not</u> cause awareness. The single effective pulse also elicits further cortical activation called <u>later evoked potentials</u>. This further activation lasts for about 500 msecs and only <u>then</u> there exists awareness. It is remarkable that there exists a subjective referral of the timing of awareness back to the time of the EP response. If a single pulse becomes weaker so that the EP remains but the lateral EPs disappear, the subject responds feeling nothing. Such a weak stimulus, namely a stimulus which does <u>not</u> elicit a conscious experience, is sometimes called <u>subliminal</u>. By the way, the effect of anaesthesia is to eliminate the later EPs.

The situation is similar with other sensory modalities, including vision. Diagrams 3a and 3b illustrate brain activation during the unconscious and conscious processing of a written word. A word presented for 32 msecs (under appropriate masked conditions) provides a subliminal stimulus, i.e. it is not consciously seen. The same word presented

for a longer period becomes visible but the person becomes aware of this word only after about 400 msecs. Functional MRI and magnetoencephalography indicate that a subliminal stimulus yields brain activation which is localized both in space and in time.
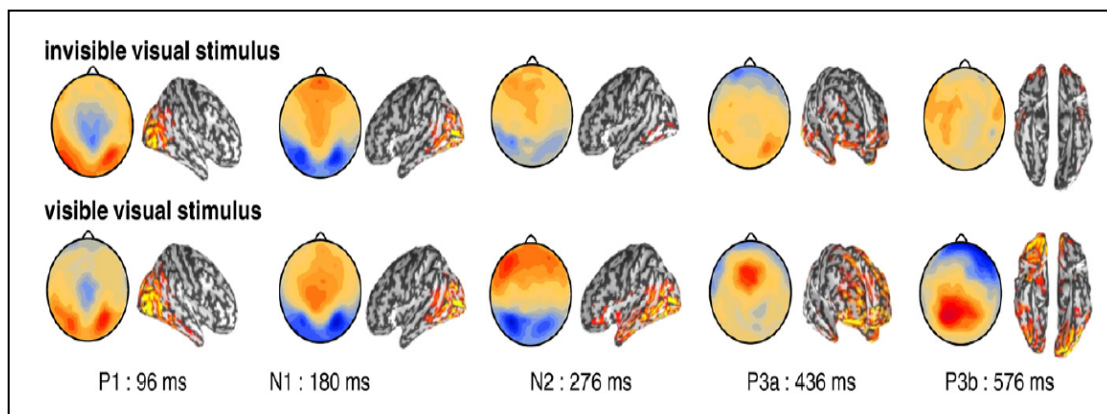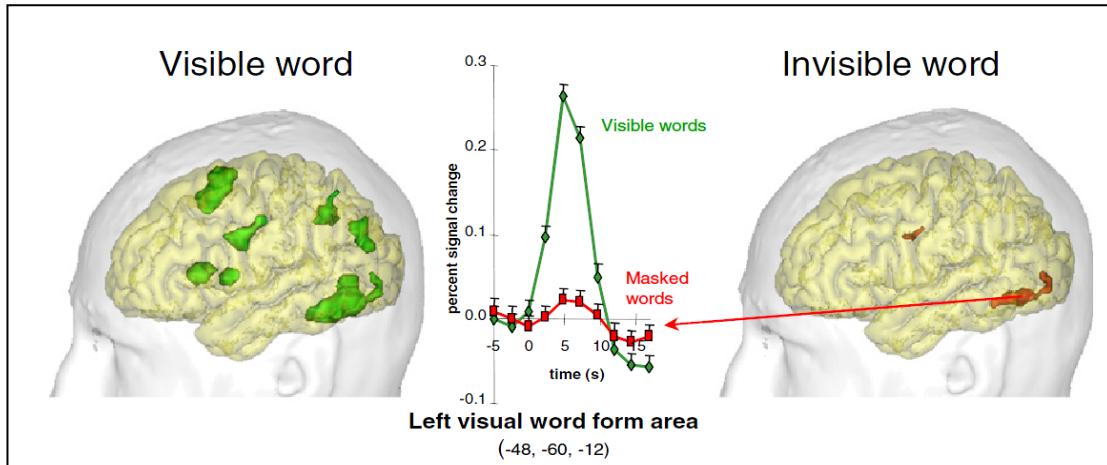
Diagrams 3a and 3b

The elucidation of the precise neuronal mechanisms responsible for a conscious experience remains open. Several neuroscientists, including the late Francis Crick, have suggested that the critical event is a global, sustained, synchronized neuronal activity perhaps around the 40 Hertz range. Changeux and Dehaene have proposed that the so-called pyramidal cells, with their long range axons which are particularly dense in the prefrontal and parietal regions, play an important role in broadcasting and synchronizing this activity.

6. Mathematics and Imaging

The images produced by PET, SPECT and functional MRI are based on the solution of a particular class of mathematical problems, called <u>inverse problems</u>. The situation is similar with MRI and Computerized Tomography (CT). Allan Cormack, in his Nobel prize speech in 1979 said: "It was obvious that the problem of computerized tomography was purely a mathematical problem". The inverse problems of MRI and functional MRI are based on the inverse Fourier transform, whereas the inverse problems of CT and PET are based on the inverse Radon transform. The inverse problem of SPECT is substantially more complicated and was solved only in 2004 using a mathematical technique introduced earlier by the eminent mathematician Israel Gelfand and the speaker. In spite of their enormous significance, the functional imaging techniques of PET, SPECT and functional MRI cannot be used for the study of the unconscious-conscious dynamics, since their time resolution is of the order of two seconds. For the study of real time processes, the most important imaging techniques are electroencephalography (EEG) and magnetoencephalography (MEG). Images obtained via these techniques are also based on the solution of certain inverse mathematical problems. However, as it was already known to Helmholtz since 1853, the solution of the relevant problems are not unique. The complete characterization of this non-uniqueness of this inverse problem was finally achieved in 2010 (A.S. Fokas, Electromagnetoencephalogoraphy, Interface Journal of the Royal Society). These new results suggest that simultaneous EEG and MEG measurements can yield information for two of the three scalar functions defining the neuronal current.

It should be emphasized that the brain <u>deconstructs</u> and <u>reconstructs</u>. This is the reason why Gerald Edelman writes that "every act of perception is an act of creation". Although there is no doubt that the brain is a creative machine par excellence, perhaps a less poetic but more accurate aphorism would be

perception is achieved through the solution of inverse problems.

Of course, the inverse problems solved by the brain are exceedingly more complicated than the mathematical inverse problems mentioned earlier. Indeed, for the solution of a mathematical inverse problem, the computer follows well-defined algorithmic steps. The brain works in a very different manner: Working within a dynamic environment which

continuously changes, the brain is willing to sacrifice accuracy in order to achieve its main goal which is the solution to the following inverse problem:

decode the meaning of infinitely many conflicting data.

Perhaps the defining property of the human brain is its amazing ability for <u>abstraction</u>. But among the sciences, the apotheosis of abstraction occurs in Mathematics. This deep relation between Mathematics and the function of the brain perhaps explains both the crucial role of Mathematics in the search for truth, as well as the inherent aesthetics found in mathematical structures. Here is an example: Before the emergence of the mathematical theory of chaos, and before it was understood that branches of trees, snowflakes, the bronchi, and a myriad of other natural forms are examples of fractal geometry, the brain of the painter Jackson Pollock, via abstraction, reconstructed this geometry and enabled Pollock to express it in his paintings. Indeed, the mathematical analysis a few years ago of some of the paintings of Pollock, clearly established that a basic characteristic of these painting is precisely their fractality.

The brain employs different types of abstraction. In visual perception, the most elemental abstraction is the processing of just lines and colour. The deconstruction of an image to these Platonic ideal elements is best illustrated in the paintings of Piet Mondrian. Pollock's paintings illustrate a more complex form of abstraction which is based on top down, Gestalt processes. Hence, Mondrian and Pollock reveal the beauty associated with two fundamental types of abstraction employed by the brain during visual processing.

Every single moment we come closer and closer to the truth, which of course we know that we will never reach. The closer we come to the truth, the higher level of aesthetics we reach, thus the deeper we can appreciate the verses of the poet John Keats in his poem ***Ode on a Grecian Urn***,

beauty is truth, truth beauty,
that is all ye know on earth,
and all ye need to know.